A Cube Root Method for
Recalculation of Kepler's Third Law

William R. Livingston
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Correspondence: bingston@gmail.com

ABSTRACT

I will show that all mathematical conclusions within Kepler's third law of planetary motion can be derived from the cube root of orbit periods. The inverse of the cube root determines orbit velocity. The square of the cube root determines the orbit radius. I will use third law calculations to show that any change of velocity has a cubic effect on the duration of an orbit.

INTRODUCTION

This work is the result of a search through the scientific literature for precise velocity and orbit radius values for each planet. Preliminary research indicated that the numerical values associated with Kepler's third law have historically been presented as approximations. My response was to search for a mathematical method that would allow velocities and orbit radiuses to be calculated more precisely. The cube root method I describe exceeds my original goal because it allows the velocities of all planets to be synchronized to one another.

I have avoided scientific symbols, and I have shown all of my mathematical operations, so that the concepts presented here are more accessible to students of science. The rudimentary calculations I have provided may be helpful to students who experience difficulty in understanding basic orbital concepts. I have attempted to write in a style that can be read, understood, and challenged by high school students who speaks English. I begin with a brief description of the cube root method; I give examples of how the method can be used; and I offer speculation regarding Kepler's belief that planetary orbits are propelled by the Sun.

Throughout this monograph I speak theoretically about the doubling of a planet's velocity. I do not mean to imply that the velocity of a planet can actually double. The terminology I have chosen is only intended to make the general theory, and associated math, easier to present and understand.

Johannes Kepler discovered that the square of a planet's orbit period is proportional to the cube of its orbit radius. This monograph describes a second method for achieving identical numerical results.

Explanatory statements for this version of the monograph appear on page 19.

The Cube Root Method for Third Law Calculations

Calculate the cube root of Mercury's orbit period

$$\sqrt[3]{87.969257175} = 4.4474421554255$$

Square the cube root

$$4.4474421554255^2 = 19.7797417258558 = Orbit\ Radius\ Number$$

Calculate the inverse of the cube root

$$\frac{1}{4.4474421554255} = 0.22484834317184 = Orbit\ Velocity\ Number$$

All numbers above correspond to a quantity of days. To make the numbers useful in terms of distance and velocity, they must be transformed into ratios and recognizable units of measurement. The tables on page 3 will show that the orbit period, the orbit radius, and the orbit velocity are defined by their mathematically perfect relationship as shown below.

The orbit radius divided by the orbit velocity is the orbit period

$$19.7797417258558 \div 0.22484834317184 = 87.969257175$$

The cube root of the orbit period divided by its inverse is the orbit radius number

$$4.44744215543 \div 0.22484834317 = 19.7797417258558$$

Throughout this document, I have avoided using the word "mean", as in mean orbit period and mean orbit velocity. Use of the word would be redundant because every number on these pages is a mean number.

Orbit Radius Numbers and AU Distance Ratios for the Planets

Table 1

Planet	Orbit Period	Cube Root of the Orbit Period	Orbit Radius Numbers (Cube Root Squared)	AU Distance Ratios
Mercury	87.969257175	4.4474421554255	19.7797417258558	0.38709903581776
Venus	224.700799215	6.07950480023533	36.9603786160844	0.72333234296196
Earth	365.256363004	7.14824227256353	51.0973675872642	1.0
Mars	686.9795859	8.82364331489275	77.8566813484516	1.52369260932058
Ceres	1680.15	11.888197699875	141.329244551314	2.76588112508831
Jupiter	4332.82012875	16.3026005219563	265.774783778488	5.20134003624741
Saturn	10,755.6986445	22.0739239442527	487.258118296653	9.53587515960605
Uranus	30,687.1530015	31.307774678788	980.176755337757	19.1825293869358
Neptune	60,190.02963	39.1899629025224	1535.85319230108	30.0573838696905
Pluto	90,553.0174125	44.9056488003837	2016.5172941834	39.4642109642848

Velocity Calculations based on the Cube Root of Orbit Periods

Table 2

Planet	Cube Root of the Orbit Period	Inverse Cube Root Velocity	Velocity Ratio Numbers	Km/sec Velocity
Mercury	4.4474421554255	0.22484834317184	1.60727043157678	47.8721246606456
Venus	6.07950480023533	0.16448708124407	1.17579350743943	35.0206986064564
Earth	7.14824227256353	0.13989453097277	1.0	29.7847354870356
Mars	8.82364331489275	0.11333187032982	0.8101236663203	24.1293191131376
Ceres	11.888197699875	0.08411703987817	0.60128898030007	17.9092332295069
Jupiter	16.3026005219563	0.06133990700767	0.43847251626735	13.0597879153579
Saturn	22.0739239442527	0.04530232153221	0.32383196982178	9.64524956338741
Uranus	31.307774678788	0.03194094790383	0.2283216340319	6.80049947560789
Neptune	39.1899629025224	0.0255167376016	0.18239982238165	5.43273046251972
Pluto	44.9056488003837	0.0222689133041	0.1591835874444	4.74124104590885

All Km/sec velocities are calibrated to Earth's velocity. The calculation for Earth's velocity is shown on page 15.

Every number presented here is a mean number. The quantity of significant figures is required so that cube roots can be used to reconstitute their source numbers. All calculations can be performed accurately in reverse order. Rounding of numbers is not permitted.

Inverse Cube Root Velocities Compared to NASA/JPL Ephemerides

Table 3

Planet	Inverse Cube Root Ratios	Km/sec Velocity	Velocities as provided by NASA / JPL	NASA Difference
Mercury	1.60727043157678	47.8721246606456	47.362	-0.51012
Venus	1.17579350743943	35.0206986064564	35.0214	+0.00070
Earth	1.0	29.7847354870356	29.7859	+0.00116
Mars	0.8101236663203	24.1293191131376	24.1309	+0.00158
Ceres	0.60128898030007	17.9092332295069		
Jupiter	0.43847251626735	13.0597879153579	13.0697	+0.00992
Saturn	0.32383196982178	9.64524956338741	9.6624	+0.01715
Uranus	0.2283216340319	6.80049947560789	6.8352	+0.03470
Neptune	0.18239982238165	5.43273046251972	5.4778	+0.04507
Pluto	0.1591835874444	4.74124104590885	4.656	-0.08524

There appears to be consistency in NASA's calculations for the planets Venus to Neptune, while NASA's calculations for Mercury and Pluto appear to be anomalous. The discrepancy for Mercury is greater than 1%.

Using Tables 1 and 2 to Prove the Orbit Period of any Given Velocity

Exercise: Find the orbit radius of Mars if its velocity is 24.12931 km/sec

Divide the target velocity by Earth's velocity to produce the Velocity Ratio Number for Mars

$$24.1293191131376 \div 29.7847354870356 = 0.8101236663203$$

Multiply by Earth's Inverse Cube Root Velocity to find the Inverse Cube Root Velocity for Mars

$$0.08101236663203 \times 0.13989453097277 = 0.11333187032982$$

Find the cube root of the orbit period

$$\frac{1}{0.11333187032982} = 8.82364331489279$$

Square the cube root to find the Orbit Radius Number for Mars

$$8.82364331489275^2 = 77.8566813484522$$

Divide by Earth's Orbit Radius Number to find the AU Distance Ratio for Mars

$$77.8566813484516 \div 51.0973675872642 = 1.5236926093206\ AU$$

Return to the cube root to find the orbit period

$$8.82364331489275^3 = 686.9795859\ days$$

Reiteration: Any given velocity corresponds to a specific orbit radius value and a specific orbit period value. Calculations of Mass and Gravity are not required. All calculations are time based.

Doubling the Velocity of a Planet, and Reducing the Orbit Period by the Factor 8

Warning: These calculations are not being compared to the orbit of Mercury. Do not be confused by numbers on this page that are very similar to numbers found in the tables.

Exercise: Double the velocity of Mars

$$24.1293191131376 \times 2 = 48.258638226274$$

Divide by Earth's km/sec velocity to produce the new Velocity Ratio Number

$$48.258638226216 \div 29.784735487 = 1.62024733264059$$

Multiply by Earth's Inverse Cube Root Velocity to find the Inverse Cube Root Velocity of the new orbit period

$$1.62024733264055 \times 0.13989453097277 = 0.22666374065964$$

Find the cube root of the new orbit period

$$\frac{1}{0.22666374065962} = 4.4118216574464$$

Square the cube root

$$4.41182165744651^2 = 19.4641703371131$$

Divide by Earth's Orbit Radius Number to find the AU Distance Ratio for the new velocity

$$19.4641703371141 \div 51.0973675872642 = 0.38092315233015\ AU$$

Return to the cube root to find the orbit period of the new velocity

$$4.41182165744651^3 = 85.8724482375072\ days$$

Divide the true orbit period of Mars by the new double-velocity orbit period to see the Factor 8 difference

$$686.9795859 \div 85.8724482375072 = 8$$

Reiteration: Doubling the velocity of a planet causes the orbit period to be reduced by the factor 8.

The Effect of a Velocity Change on AU Distances and Orbit Periods

This table shows that any velocity change has a squared effect on the radius of the orbit, and a cubed effect on the orbit period.

Table 4

Velocity Change Factor	AU Distance (Orbit Radius) Change Factor	Orbit Period Change Factor
X	X^2	X^3
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
Etc.	Etc.	Etc.

The example below shows that whole numbers are not required for velocity change calculations.

Table 5

Velocity Change Factor	AU Distance (Orbit Radius) Change Factor	Orbit Period Change Factor
X	X^2	X^3
4.7692	22.7452	108.4767

Examples of Double Velocity Corresponding to a Factor 8 Energy Requirement

Measuring Wind Speeds for Small Turbine Sites - "Wind power (watts) is proportional to the wind speed cubed. If the wind speed doubles, the available wind power increases by a factor of eight" **Oregon State University Extension Service**

Steam Power on Canals - "It appears, therefore, in order to double the speed (of a river boat) the propelling power must be increased eight times". **Scientific American, 23 October 1869**

Cube Law Torque Load - "Power requirement increases with the cube of the increase in speed. Therefore, there is a large increase in power requirement for a small increase in speed. For example: To double the machine speed would require eight times the power to drive it. Conversely, if the speed of the machine is halved, the power requirement is reduced to 1/8 of the original power". **Honeywell Variable Frequency Drive Application Guide**

Elements of Hydraulics: A Textbook for Secondary Technical Schools – Theoretic Hydraulics section – "Hence the horse powers of jets of the same cross-section vary as the cubes of their velocities. For example, if the velocity of a jet be doubled, the cross-section remaining the same, the horse power is made eight times as great".

Fan System Effect – "therefore, if it is required to double the air flow through a system, the fan must be capable of providing twice the volume flow rate at four times the original pressure! AND EIGHT TIMES THE FAN MOTOR POWER!" **PDH Course M213 – www.pdhcenter.com**

Plasma Propulsion Research at NASA Marshall Space Flight Center – The transit time to a destination scales approximately inversely with the cube root of the specific power…Consequently, reducing a trip time by half requires roughly an eight-fold increase in specific power. **NASA Technical Reports Server (NTRS)**

If Planetary Orbits are Propelled by the Sun

In the wind power example above, the doubling of velocity corresponds to an eightfold increase of available energy. The other examples show that the doubling of velocity or speed requires eight times the power to drive the system. If this same eightfold increase can be applied to orbital physics, and if the sun is the source of orbital power as Johannes Kepler wanted to believe, the quantity of energy per orbit would always be unity for the following reason. While eight times the power would be required to double a planet's velocity, the orbit period would simultaneously be reduced by the factor 8. Therefore, every velocity increase would be balanced with a commensurately shorter orbit so that the quantity of propulsive energy per orbit is always constant.

"The power that moves the planets resides in the body of the Sun" Johannes Kepler, Astronomia Nova, 1609, translated by William H. Donahue, 1992

APPENDIX

The following pages contain tables, exercises, and speculation that may be useful to students of science.

The Classic Energy Equation
Based on the Doubling of Velocity and Quadrupling of Energy

Table 6

MASS	VELOCITY	ENERGY
1 Unit 1 Unit 1 Unit 1 Unit M	1 Unit 2 Units 4 Units 8 Units C	1 Unit 4 Units 16 Units 64 Units MC^2
If an imaginary MASS is moving at light speed in a "thought experiment" the velocity is "C", and this table is compatible with the equation $E = MC^2$		

The Energy Equation Applied to Doubling of Wind Velocity

Table 7

MASS	VELOCITY	WIND ENERGY
1 Wind 1 Wind 1 Wind 1 Wind M	1 Unit 2 Units 4 Units 8 Units C	1 Unit 8 Units 64 Units 512 Units MC^3
If an imaginary WIND is moving at light speed in a "thought experiment" the velocity is "C", and this table is compatible with the equation $E = MC^3$		

We can see that wind energy does not comply with the classic energy equation at the top of the page. The classic equation allows for a quadrupling of energy when velocities double. But it is well known that the doubling of wind velocity increases energy potential by the factor eight. If this same factor eight increase can be applied to the energy of an "orbital wind" caused by the Sun, the energy required to move the planets in their orbits would comply with the equation $E = MC^3$.

Calculating an Orbit Located Near the Surface of the Sun

Astronomical Unit	149,597,870.7 km	Earth's orbit radius number	51.0973675872642
Solar Radius	696,000 km	Earth's inverse cube root velocity	0.13989453097277
Solar Radius in AU's	0.00465247264	Solar circumference	4,373,096.973797 km

Multiply the Solar Radius AU number by Earth's Orbit Radius Number to find the Orbit Radius Number corresponding to the surface of the Sun

$$0.00465247264 \times 51.0973675872642 = 0.23772910467577$$

The square root of the Orbit Radius Number is the Cube Root of the Orbit Period

$$\sqrt{0.23772910467577} = 0.4875747170186$$

Find the Inverse Cube Root of the Orbit Period

$$\frac{1}{0.4875747170186} = 2.05096770832326$$

Find the Velocity Ratio Number corresponding to an orbit near the solar surface

$$\frac{2.05096770832326}{0.13989453097277} = 14.660814072299$$

Multiply by Earth's km/sec velocity to find the velocity required for an orbit to exist near the surface of the Sun

$$14.660814072299 \times 29.7847354870356 = 436.668469168027\ km/second$$

Divide the solar circumference by the velocity to find the orbit period

$$\frac{4{,}373{,}096.973797\ km}{436.668469168027\ km/sec} = 10{,}014.6845549186\ seconds$$

Therefore, any object able to orbit just above the surface of the Sun would complete its orbit in $\sim$10,014 seconds, or $\sim$2.78 hours.

Calculating an Orbit Located Near the Surface of the Earth

Earth's Equatorial Radius	6378 km	Earth's orbit radius number	51.0973675872642
Earth's Radius in AU's	0.00004263429667	Earth's inverse cube root velocity	0.13989453097277
Sun/Earth Mass Ratio	333,000		

Multiply Earth's Radius in AU's by the Sun/Earth Mass Ratio

$$0.00004263429667 \times 333{,}000 = 14.19722079111$$

Find the corresponding Orbit Radius Number

$$14.19722079111 \times 51.0973675872642 \; = 725.440609480895$$

The square root of the Orbit Radius Number is the Cube Root of the Orbit Period

$$\sqrt{725.440609377766} = 26.9340047055928$$

Find the Inverse Cube Root Velocity of an orbit near the surface of the Earth

$$\frac{1}{26.9340047055928} = 0.03712778737996$$

Find the Velocity Ratio Number of the orbit

$$\frac{0.03712778737996}{0.13989453097277} = 0.26539841923616$$

Find the velocity required for an orbit to exist near the surface of the Earth

$$0.26539841925502 \times 29.7847354870356 \; = 7.90482171562612 \; km/second$$

Divide Earth's circumference by the velocity to find the orbit period

$$\frac{40{,}075 \; km}{7.90482171617871 \; km/sec} = 5{,}069.69055618046 \; seconds$$

Therefore, any object able to orbit just above the surface of the Earth would have an orbit period of 5,069.69 seconds, or $\sim 1.4 \; hours$. The velocity would be $\sim 28{,}457 \; km/hour$.

Using Tables 1 and 2 to Find the AU Distance of an Orbit Velocity around the Sun

Exercise: Find the AU distance of an object traveling at 1.023 km/sec

Divide the target velocity by Earth's km/sec velocity to produce the Velocity Ratio Number of the object

$$1.023 \div 29.7847354870356 = 0.03434645241168$$

Multiply by Earth's Inverse Cube Root Velocity to find the Inverse Cube Root Velocity of the orbit period

$$0.03434645241168 \times 0.13989453097277 = 0.00480488085071$$

Find the Cube Root of the Orbit Period

$$\frac{1}{0.00480488085071} = 208.121706046477$$

Square the cube root

$$208.121706046477^2 = 43{,}314.6445276963$$

Divide by Earth's Orbit Radius Ratio Number to find the AU Distance of 1.023 km/sec

$$43{,}314.6445276963 \div 51.0973675872642 = 847.688375604156\ AU$$

Return to the cube root to find the orbit period in days

$$208.121706046477^3 = 9{,}014{,}717.71589991\ days$$

Find the orbit period in sidereal years

$$\frac{9{,}014{,}717.71589991}{365.256363004} = 24{,}680.5220359739\ years$$

Therefore, an object orbiting the Sun at 1.023 km/sec would have a mean orbit radius of 847.688 AU. The object would travel around the Sun in 24,680.522 sidereal years. It is also true that Earth's Moon has a mean orbital velocity of 1.023 km/sec and an orbit period much shorter than 24,000 years. So there must be a significant difference in the physical laws that govern these two types of orbit. That difference is explained below.

The Orbit Radius of a Specific Velocity Varies According to the Mass of the Body Being Orbited.

Mean velocity of the Moon 1.023 km/sec

Orbit radius of the Moon 384,399 km

Orbit radius of the Moon in AU's 0.00256954860521 AU

AU distance of 1.023 km/sec (Solar) 847.688 AU

Ratio of the two AU distances: 847.688 ÷ .00256954860521 = 329,897.631934743

The AU distances of the Moon and the theoretical object orbiting the Sun differ by a factor of nearly 330,000, which closely resembles the official Sun/Earth Mass ratio of 333,000. This is an indication that any given velocity is paired with a specific AU distance according to the Mass of the larger body. The accuracy of the following calculations illustrate that this technique is used by scientists to determine the Mass of planets.

Table 8

Moons of Jupiter	Km Orbit Radius	AU Equivalent Orbit Radius	Velocity	The Orbit Radius if Orbit was Solar
Io	421,700 km	0.00281889038946	17.334 km/sec	2.95250 AU
Europa	670,900 km	0.00448468950033	13.740 km/sec	4.69909 AU
Ganymede	1,070,400 km	0.00715518205568	10.880 km/sec	7.49427 AU
Callisto	1,882,700 km	0.01258507217509	8.204 km/sec	13.18063 AU

Table 9

Moons of Jupiter	Comparison of Solar AU distances and Jovian AU distances	Sun/Jupiter Mass Ratio
Io	$2.9525\ AU \div 0.00281889038946\ AU = 1{,}047.3979446$	1,047 to 1
Europa	$4.69909\ AU \div 0.00448468950033\ AU = 1{,}047.8072115$	
Ganymede	$7.49427\ AU \div 0.00715518205568\ AU = 1{,}047.3905404$	
Callisto	$13.18063\ AU \div 0.01258507217509\ AU = 1{,}047.322559$	

These two tables show that any specific orbit velocity is paired with an AU distance that varies according to the Mass of the body being orbited. Velocity 17.334 around the Sun belongs to AU distance 2.9525. The same velocity around Jupiter belongs to AU distance 0.0028188946. The orbit radius of each moon is smaller by a factor of $\sim$1,047 because Jupiter's Mass ratio compared to the Sun is that much less. Therefore, the Mass of the Sun can be compared to the Mass of a planet by determining the velocity and orbital radius of each moon.

Earth's km/sec Velocity as the Standard Velocity

The Astronomical Unit of distance

149,597,870.7 km

Formula for the circumference of a circle

$2\pi r$

Calculation for the circumference of Earth's orbit

$$2 \times \pi \times 149{,}597{,}870.7 = 939{,}951{,}143.167592\ km$$

Divide the orbit circumference by the quantity of seconds per sidereal year

$$939{,}951{,}143.1676\ \div\ 31{,}558{,}149.76355\ = 29.784\ 735\ 487\ 0356$$

Earth's mean orbit velocity

29.784 735 487 0356 km/sec

All km/sec velocity calculations in this monograph are calibrated to the orbital velocity of Earth. If there is ever a change in the official value of the Astronomical Unit or the sidereal year, all km/sec velocities in the tables must be corrected. Changes in the official orbit period value of any other planet will only cause a need for corrections to that particular planet in the tables.

The Orbit Velocity Formula Based on Kepler's Order of Operations

$$V \propto \frac{1}{\sqrt[2]{orbit\ radius}}$$

This classic orbit velocity formula is based on the orbit radius of each planet. The formula indicates that velocity is directly proportional to the inverse square root of a planet's distance from the Sun.

The Inverse Cube Root Velocity Formula

$$V \propto \frac{1}{\sqrt[3]{orbit\ period}}$$

This formula is based on the inverse cube root of the orbit period, which means that it is a time calculation rather than a distance calculation. It is arguably more logical than the classic velocity formula because Kepler's third law was derived from time calculations.

A Review of Kepler's Order of Operations for the Third Law

Square the orbit period of Mercury

$$87.969257175^2 = 7{,}738.59020792129$$

Calculate the cube root

$$\sqrt[3]{7{,}738.59020792129} = 19.7797417258558 = orbit\ radius\ number$$

Table 10

Planet	Orbit Period	Orbit Period Squared	Orbit Radius Number	AU Distance Ratios
Mercury	87.969257175	7,738.59020792129	19.7797417258558	0.38709903581776
Venus	224.700799215	50,490.4491678597	36.9603786160844	0.72333234296196
Earth	365.256363004	133,412.21071491	51.0973675872642	1.0
Mars	686.9795859	471,940.951443335	77.8566813484516	1.52369260932058
Ceres	1680.15	2,822,904.0225	141.329244551314	2.76588112508831
Jupiter	4332.82012875	18,773,330.2681012	265.774783778488	5.20134003624741
Saturn	10,755.6986445	115,685,053.331299	487.258118296653	9.53587515960605
Uranus	30,687.1530015	941,701,359.33747	980.176755337757	19.1825293869358
Neptune	60,190.02963	3,622,839,666.86028	1535.85319230108	30.0573838696905
Pluto	90,553.0174125	8,199,848,962.50853	2016.5172941834	39.4642109642848

For velocities, first calculate the square root of the orbit radius

$$\sqrt[2]{19.7797417258558} = 4.4474421554255$$

Then calculate the inverse square root of the orbit radius

$$\frac{1}{4.4474421554255} = 0.22484834317184 = velocity\ expressed\ in\ days$$

The Proposed Reason that Kepler's Calculations are Successful

The first calculation produces the cube root of the orbit period to the sixth power

$$7{,}738.59020792129 \ = \ \sqrt[3]{X}^{6}$$

The second calculation produces the cube root of the orbit period to the second power

$$19.7797417258558 \ = \ \sqrt[3]{x}^{2} \ = \ orbit\ radius\ number$$

The third calculation produces the cube root of the orbit period

$$4.4474421554255 \ = \ \sqrt[3]{x}$$

The fourth calculation produces the inverse of the cube root of the orbit period

$$0.22484834317184 = \frac{1}{\sqrt[3]{x}} = inverse\ cube\ root\ velocity$$

Related Mathematical Trivia

The relationship of the inverse cube root to its source number

$$0.22484834317184 \div 0.22484834317184 \div 0.22484834317184 = 4.4474421554255$$

The relationship of the cube root to its source number

$$4.4474421554255 \times 4.4474421554255 \times 4.4474421554255 = 87.969257175$$

References:

NASA/JPL Horizons Web-interface for ephemerides – easily located online by search engine

The IAU 2009 System of Astronomical Constants – easily located online by search engine

NASA Data Sheets of the Planets, Sun, and Moon – easily located online by search engine

References for scholars:

Harmonies of the World, 1619, Johannes Kepler - This introduces Kepler’s third law of planetary motion.

Astronomia Philolaica, 1645, Ismael Bullialdus - This contains what is believed to be the first mention of an inverse square law.

Mathematical Principles of Natural Philosophy, 1687, Isaac Newton - This contains references to the work of Kepler and Bullialdus.

Mathematical Elements of Natural Philosophy, Confirmed by Experiments: or an Introduction to Sir Isaac Newton’s Philosophy, 1722, W. J. s’Gravesande, translated by Desaguliers, 1747 – It is generally believed that the experiments in this book confirm that the doubling of velocity corresponds to a quadrupling of energy, a result conflicting with Newton’s opinion; and favoring the opinion of Leibniz.

The previous version of this paper dated 15 July 2014 contained an error on the reference page.

An important clarification has been added to the bottom of page 3.

A paragraph has been added to page 8.

Page 20 has been removed due to redundancy.

A mathematical operation was moved from page 18 to page 2.

The Author suggests that orbit velocity exists, even where planets do not.